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Answers Key:

ANSWER ANY TWO

1. $\int \frac{3x^2+6x+1}{(x+3)(x^2+1)}dx = \int \left[\frac{1}{(x+3)} \frac{2x}{(x^2+1)}\right]dx$
   $\int \frac{dx}{(x+3)} + \int \frac{2x}{(x^2+1)}dx$
   $= \log|x+3| + \log|x^2+1| + c$
   $= \log\left|(x+3)\left(x^2+1\right)\right| + c$
   $= \log\left|x^3+3x^2+x+3\right| + c$
   பகுதி பின்னமாக்கல் முறையில்
   $\frac{3x^2+6x+1}{(x+3)(x^2+1)} = \frac{A}{(x+3)} + \frac{Bx+C}{(x^2+1)}$
   $\Rightarrow \frac{3x^2+6x+1}{(x+3)(x^2+1)} = \frac{1}{(x+3)} + \frac{2x}{(x^2+1)}$.
2. I= $\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}}$ dx என்க. .....(1)
   I= $\int_2^5 \frac{\sqrt{2+5-x}}{\sqrt{2+5-x}+\sqrt{7-(2+5-x)}}dx$ $\left[\because \int_0^a f(x)dx \int_0^a (a-x)dx\right]$
   I= $\int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x}+\sqrt{x}}dx$ ....(2)
   (1)+(2)⇒
   2I= $\int_2^5 \left[\frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}} + \frac{\sqrt{7-x}}{\sqrt{7-x}+\sqrt{x}}\right]dx$
   = $\int_2^5 \left[\frac{\sqrt{x}+\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\right]dx$
   = $\int_2^5 dx = [x]_2^5 = 3$
   ∴ I= $\frac{3}{2}$
3. $\int_a^b f(x)dx = \lim_{n\to\infty h\to 0} \sum_{r=1}^{n} f(a+rh)$ ...(1)
   இங்கு a=1, b=2, h= $\frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$ மற்றும் f(x)=2x+1
   f(a+rh)= $f\left(1+\frac{r}{n}\right)$
   =2$\left(1+\frac{r}{n}\right)$+1
   2+ $\frac{2r}{n}$+1
   f(a+rh)=3+ $\frac{2r}{n}$
   இதை (1) -இல் பிரதியிட, நாம் பெறுவது
   $\int_1^2 (2x+1)dx = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(3+\frac{2r}{n}\right)$
   = $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(3+\frac{2r}{n}\right)$
   = $\lim_{n\to\infty} \left[\frac{3}{n}\sum_{r=1}^{n} 1 + \frac{2}{n^2}\sum_{r=1}^{n} r\right]$
   = $\lim_{n\to\infty} \left[\frac{3}{n}n\frac{2}{n^2}\frac{n(n+1)}{2}\right]$
   =3 + $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)$
   ∴ $\int_1^2 (2x+1)dx$
   =3+1=4

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4. $\int (x+b)dx$
   $f(x) = \frac{x^2}{2} + bx + c \ldots (1)$
   f(1) =5 என்பதை (A) ல் பிரதியிட
   5 = $\frac{1}{2}$+b+c
   5(2) = 2b+2c+1
   2b+2c =9 .....(1)
   f(2) =13 என்பதை (1) ல் பிரதியிட
   13 = $\frac{4}{2}$ +2b+2c
   13 = 2+2b+c
   2b+c =11 .....(2)
   (1),(2) ஆகியவற்றைத் தீர்க்க
   c=-2, 2b=13, b= $\frac{13}{2}$
   f(x)= $\frac{x^2}{2} + \frac{13}{2}$x-2
5. $\int \left(e^{x\log a} + c^{a\log a} - e^{n\log x}\right)dx$
   = $\int \left(e^{\log a^x} + c^{\log a^a} - e^{\log x^n}\right)dx$
   = $\int \left(a^x + a^a - x^n\right)dx$
   = $\frac{a^x}{\log a} + a^a x - \frac{x^{n+1}}{n+1} + c$
6. $\int_1^4 f(x) = \int_1^2 f(x)dx + \int_2^4 f(x)dx$
   = $\int_1^2 (4x+3)dx + \int_2^4 (3x+5)dx$
   $= \left(\frac{4x^2}{2}+3x\right)_1^2 + \left(\frac{3x^2}{2}+5x\right)_2^4$

$= (2x^2 + 3x)_1^2 + \left(\frac{3x^2}{2} + 5x\right)_2^4$

$= 2(2^2) + 6 - (2+3) + \left[\frac{3(16)}{2} + 20\right] - \left[\frac{3(2^2)}{2} + 10\right]$

= (8+6) - (2-3) + (24+20) - (6+10)

= 14-5+44-16

= 58 - 21 = 37

$f(x) - \begin{cases} x^2 e^{-2x}, x \geq 0 \\ 0, \text{ மற்றொருவிதமும்} \end{cases}$

7.

$\therefore \int_0^\infty f(x)dx = 0 \int_0^\infty x^2 e^{-2x} dx$

இங்கு n = 2 and a = 2

$\left[\because \int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}\right]$

$\therefore \int_0^\infty f(x)dx = \frac{2!}{2^{2+1}} = \frac{2!}{2^3}$

$= \frac{2 \times 1}{2 \times 2 \times 2} = \frac{1}{4}$

$\therefore \int_0^\infty f(x)dx = \frac{1}{4}$

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8. $\int \frac{3x^2+2x+1}{x} dx = \int (3x + 2 + \frac{1}{x})\text{dx}$

$= \frac{3x^2}{2} + 2x + \log|x| + c$

9. $\int \frac{1}{\sqrt{x+2}-\sqrt{x-2}} dx$

$= \int \frac{\sqrt{x+2}+\sqrt{x-2}}{4} dx$

$= \frac{1}{6}\left\{(x+2)^{\frac{3}{2}} + (x-2)^{\frac{3}{2}}\right\} + c$

காரணக்காரிய முறையில்,

$\frac{1}{\sqrt{x+2}-\sqrt{x-2}} = \frac{1}{\sqrt{x+2}-\sqrt{x-2}} \times \frac{\sqrt{x+2}-\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}}$

$= \frac{\sqrt{x+2}-\sqrt{x-2}}{4}$.

10. $\int (2\cos x - 3\sin x + 4\sec^2 x - 5cosec^2 x)dx$

= 2sin x+3cos x+4tan x+5cot x+c

11. $\Gamma\left(\frac{9}{2}\right) = \frac{7}{2}\Gamma\left(\frac{7}{2}\right)$

$= \frac{7}{2}\Gamma\left(\frac{5}{2}\right)$

$= \frac{7}{2} \times \frac{5}{2} \times \Gamma\left(\frac{3}{2}\right)$

$= \frac{7}{2} \times \frac{5}{2} \times \left(\frac{3}{2}\right) \times \Gamma\left(\frac{1}{2}\right)$

$= \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2}\sqrt{\pi}$

$= \frac{105}{16}\sqrt{\pi}$

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